Many-body phases of cold atoms in optical lattices and magnetic microtraps

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• Introduction. Controlled studies of strongly correlated atomic systems in optical lattices

• Superfluidity of fermionic atoms

• Two component mixtures
  Quantum magnetism of spin-$\frac{1}{2}$ bosons

• Spin-1 bosons in optical lattices

• Fragmentation of quasi one-dimensional BEC in magnetic microtraps

• Conclusions
Bose–Einstein condensation of atomic gases

Anderson et al., Science (1995)

Ultralow density condensed matter system

\( n \sim 10^{14} \text{ cm}^{-3} \)

\( T_{\text{BEC}} \sim 1 \mu \text{K} \)

Interactions are weak and can be described theoretically from first principles
Strongly interacting bosons in optical lattices

Superfluid to insulator transition

Greiner et al., Nature (02)

Following theoretical suggestion by Jaksch et al.
PRL (98)
BCS superfluidity of fermionic atoms

Cooling Fermi gases to quantum degeneracy $T = 0.1 \ldots 0.5 T_F$

$^4\text{K}$: De Marco and Jin, Science 285, 1703 (99)

$^6\text{Li}$: Truscott et al. Science 291, 2570 (01)
Schreck et al. PRA 64, 011402 (01)
Hadzibabic et al. PRL 88, 160401 (02)
O'Hara et al. Science 298, 2179 (02)

At low temperatures attractive interaction between atoms leads to pairing: BCS superfluidity

$$T_c \sim T_F e^{-\frac{1}{\sqrt{n} |a_s|}}$$

For weakly interacting gases ($\sqrt{n} |a_s| \ll 1$) transition temperature is exponentially suppressed

Strong interaction limit near Feshbach resonance
Theoretical work by M. Holland, E. Timmermans, H. Stoof, ...
Experimental observation of molecular BEC
M. Greiner et al. cond-mat/0311172
Enhancing superfluidity of fermionic atoms using optical lattices
Hofstetter, Cirac, Zoller, Lukin, Dowler, PRL 89, 220407 (02)

Optical lattices enhance interactions and reduce kinetic energy of atoms. Both enhance superfluidity.

Effective description:
Hubbard model, $V < 0$

$$\mathcal{H} = -t \sum_{\langle i, j \rangle} \hat{c}_i^\dagger \hat{c}_j + U \sum_i \hat{N}_i \hat{N}_i$$

Polarized bosonic atoms in optical lattices:
superfluid - Mott insulator transition ($t \sim \hbar \Omega_{kF}$)

Theory: Jaksch et.al. PRL 81, 3108 (98)

Experiment: Greiner et.al. Nature 415, 39 (02)  
Orgel et.al. Science 291, 2386 (01)
Enhancing superfluidity of fermionic atoms using optical lattices

\[ \mathcal{H} = -t \sum_{<i,j>} C_i^\dagger C_j + U \sum_{i} N_i \eta_i N_i \eta_i \]

\( t \gg |U| \) BCS regime \( T_c \sim T e^{-7t/U} \)

\( t \ll |U| \) Condensation of composite bosons \( T_c \sim \frac{e^2}{U} \)

Highest transition temperature for \( t \sim U \)

\[ T_c^{\text{MAX}} \propto T_F^{\text{free}} \frac{3}{\sqrt{n}} |a_s| \]

In combination with effective atomic cooling due to turning on the optical lattice

\[ T_{\text{in}}^{\text{MAX}} \propto 0.1 T_F^{\text{free}} \]
Cold atom test of high-Tc mechanism in cuprates

Cold repulsive fermions in a lattice

Effective description: Hubbard model, \( U > 0 \)

\[
\mathcal{H} = -t \sum_{\langle ij \rangle} c^\dagger_i c_j + U \sum_i n_i \uparrow n_i \downarrow
\]

- Antiferromagnetic order when lattice is completely filled (1 atom per site)
- d-wave superfluidity at fractional fillings

\[\uparrow \downarrow \] - \[\uparrow \uparrow \]
Signatures of the superfluid phase.

Interference pattern after a free expansion

Second order interference  

\[ g(r, r') = \langle n(r) n(r') \rangle_t - \langle n(r) \rangle_t \langle n(r') \rangle_t \]

Normal state

\[ g_N(r, r') = \frac{N}{w^3} \delta(r-r') p(r) - \frac{N}{w^3} \left( \frac{2\pi a}{\xi} \right)^3 \frac{2\psi(r)}{\xi} \sum \delta(r-r' + \frac{\hbar t}{m}) \]

\[ W = \frac{\hbar t}{am} \]

Superfluid state

\[ g_s(r, r') = g_N(r, r') + \frac{N}{w^3} \left( \frac{2\pi a}{\xi} \right)^3 \psi(r) \sum \delta(r-r' + \frac{\hbar t}{m}) \]

\[ \psi(r) = 2 |u(|a(r)|^2) \left( |v(a(r)|^2 \right|^2 \]

\[ Q(r) = \frac{mr}{\hbar t} \]

The peak positions at \[ r + r' = \frac{h}{m} \xi \] reflect coherent pairing at the center of mass momentum zero.
Second order interference from the BCS superfluid phase

\[ \Delta n(r, -r) \Psi_{BCS} > = 0 \]

\[ \Delta n(r, r') = n(r) - n(r') \]

\[ \langle \Delta n(r, r')^2 \rangle \]

BCS
BEC

\[ \begin{array}{c}
0.0 \\
0.2 \\
0.4 \\
0.6 \\
0.8 \\
1.0 \\
1.2 \\
1.4
\end{array} \]

\[ \begin{array}{c}
-1 \\
-0.5 \\
0 \\
0.5 \\
1
\end{array} \]
Signatures of the superfluid phase

Bragg scattering from collective mode
Hofstetter et al. PRL 89, 220407(02)

Pair of non-collinear laser beams creates atomic excitation with given frequency and momentum

In the superfluid phase

- Energy gap to quasiparticle excitations
- Sharp collective mode
"Designing" quantum magnetism in optical lattices

System

- Bosonic atoms with two trapped states

\[ \mathcal{H} = - \sum_{\langle ij \rangle} t_{ij} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} \sum_i U_{ii'} \hat{n}_i \hat{n}_{i'} \]

- Filled lattice in insulating phase
  Effective spin \( \frac{1}{2} \) system
  Spin interaction due to virtual tunneling

Tools

- Spin dependent optical potential

- Variable interaction strength between atoms in different spin states (scattering length)
  e.g. \( U_{\uparrow \downarrow} \) can be changed w.r.t. \( U_{\uparrow \uparrow} = U_{\downarrow \downarrow} \)

Spin dependent optical lattice potentials
Mandel et. al. cond-mat/0301169
Phillips et. al.
Quantum magnetism of bosons in optical lattices
XXZ magnetic systems with tunable parameters
Duan, Lukin, Demler c/m/0210564
Kuklov, Svistunov, PRL 90, 100401 (2003)

\[ \mathcal{H} = J_z \sum_{i,j} \sigma_i^z \sigma_j^z + J_\perp \sum_{i,j} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y) \]

\[ J_z = \frac{t_\uparrow^2 + t_\downarrow^2}{2U_{\uparrow\downarrow}} - \frac{t_\uparrow^2}{U_\uparrow} - \frac{t_\downarrow^2}{U_\downarrow} \]

\[ J_\perp = -\frac{t_\uparrow t_\downarrow}{U_{\uparrow\downarrow}} \]

By changing atomic and lattice properties we can manipulate

- sign of the interactions
  - ferromagnetic \( U_{\uparrow\downarrow} \gg U_{\uparrow\uparrow}, U_{\downarrow\downarrow} \)
  - antiferromagnetic \( U_{\uparrow\downarrow} \ll U_{\uparrow\uparrow}, U_{\downarrow\downarrow} \)
- anisotropy
  \[ |J_z/J_\perp| > 1 \text{ easy axis} \]
  \[ |J_z/J_\perp| < 1 \text{ easy plane} \]
Toward full phase diagram:
mean-field + quantum fluctuations
Altman, Lukin, Demler, Hofstetter
New J. Physics 5, 113 (03)

Antiferromagnetic case $U_{uu} = U_{dd} = 2U_{ud}$
cond-mat/0306683

- Magnetic phases
- Melting may result in many interesting phases
Probing spin order of bosons

Ferromagnetic insulating state of spin-$\frac{1}{2}$ bosons

$|\Phi_0\rangle = \prod_i (a_{i\uparrow}^+ + e^{iq_i} a_{i\downarrow}^+ ) |0\rangle$

1st order coherence

2nd order coherence

$G(r-r')$

Spin order at wavevector $q$

$|\Phi_q\rangle = \prod_i (a_{i\uparrow}^+ + e^{iq\cdot r_i} a_{i\downarrow}^+ ) |0\rangle$

$\propto q$

$\propto G$

$\propto r-r'$
Designing exotic phases

Optical lattice in 2 or 3 dimensions: polarization and frequencies may be different for different directions

Exactly solvable lattice model on a honeycomb lattice by Kitaev

\[ \mathcal{H} = J_x \sum_{ij \in e} 6_i^x 6_j^x + J_y \sum_{ij \in e} 6_i^y 6_j^y + J_z \sum_{ij \in e} 6_i^z 6_j^z \]

- Can be created with 3 sets of standing wave light beams
- Has non-trivial topological order, anyons, ...
Spinor condensates in optical traps

Spin symmetric interaction of F=1 atoms

\[ U(r_1 - r_2) = \delta(r_1 - r_2) (W_0 + W_2 \vec{S}_1 \cdot \vec{S}_2) \]

\[ W_2 = \frac{4\pi \hbar^2}{3M} (a_2 - a_0) \]

Antiferromagnetic interaction for \( W_2 > 0 \)

\[ ^{23}\text{Na} \quad a_0 = 46 \pm 5 \text{ a}_B \]
\[ a_2 = 52 \pm 5 \text{ a}_B \]

Spin structure in a single trap

\[ \hat{a}_x + i \hat{a}_y = \hat{a}_{1} \quad \hat{a}_z = \hat{a}_0 \]

Mean-field:

\[ (n_x a_x^+ + n_y a_y^+ + n_z a_z^+) \quad 10> \]

Ho, PRL 81, 1472 (98)

Ohmi, Machida, JPSJ 67, 1822 (98)

Beyond mean-field: spin singlet ground state

\[ (\hat{a}_x^+ + \hat{a}_y^+ + \hat{a}_z^+)^{N/2} \quad 10> \]

Law, Pu, Bigelow, PRL 81, 5257 (98)

Ho, Yip, PRL 89, 4031 (00)

Castin, Hergog, cond-mat/0012040 (01)


cond-mat/0005001
Spin $F=1$ atoms in optical lattices
Effective Hubbard type Hamiltonian

$$H = -t \sum_{\langle ij \rangle m} a_{im}^+ a_{jm} + U_0 \sum_i n_i (n_i - 1) + U_2 \sum_i \hat{S}_i^2$$

Symmetry constraints: $N_i + S_i = \text{even}$, $\hat{S}_i \leq N_i$

General phase diagram for $d > 1$

Nematic phase breaks spin rotational symmetry but not time reversal symmetry

$$|N\rangle = \prod_i \left( n_x a_{iX}^+ + n_y a_{iY}^+ + n_z a_{iZ}^+ \right)^N |0\rangle$$

Spin singlet phase

$$|S\rangle = \prod_i \left( a_{iX}^{+2} + a_{iY}^{+2} + a_{iZ}^{+2} \right)^{N/2} |0\rangle$$

Zhou, Demler, PRL 88, 163001 (02)
ImamBekov, Lukin, Demler, cond-mat/0306204
Nematic insulating phase for $N = 1$

Effective $S = 1$ spin model

$$\mathcal{H} = -J_1 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - J_2 \sum_{\langle ij \rangle} (\mathbf{S}_i \cdot \mathbf{S}_j)^2$$

$$J_1 = \frac{2t^2}{U_0 + U_2} \quad J_2 = \frac{2t^2}{3(U_0 + U_2)} + \frac{4t^2}{3(U_0 - 2U_2)}$$

Two site problem

<table>
<thead>
<tr>
<th>$S_{\text{tot}}$</th>
<th>$\mathbf{S}_i \cdot \mathbf{S}_j$</th>
<th>$(\mathbf{S}_i \cdot \mathbf{S}_j)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
<td>4</td>
</tr>
</tbody>
</table>

Singlet state is favored when $J_2 > J_1$

Can not have singlets on neighboring bonds

Classical nematic state is a superposition of $S_{\text{tot}} = 0$ and $S_{\text{tot}} = 2$ on each bond
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Signatures of Singlet and Nematic Phases
Bragg scattering from collective spin excitations

Spin singlet phase

Five-fold degenerate massive magnons

Nematic phase

Two spin waves
Three \((2+1)\) massive amplitude modes

Particle density

Use small mag. field to orient nematic
BEC in quasi one-dimensional magnetic microtrap

Condensate fragmentation in magnetic microtraps
Leinhard et al., PRL 89, 040401 (2002)
Fortagh et al., PRA 66, 041604 (2002)
Singlet and nematic insulating phases for $N=2$

$S_i = 0$ and $S_i = 2$ states are allowed.

$U_2 S_i^2$ favors $S_i = 0$.

Spin exchange ($\sim t^2/U_0$) allows "scattering".

$S_i = 0$  $S_j = 0 \Rightarrow S_i = 2$  $S_j = 2$  $S_i + S_j = 0$

First order singlet - nematic transition $\frac{2t^2}{U_0 U_2} \approx \frac{1}{2}$

For even filling factors $S-N$ transition

$\frac{2N^2t^2}{U_0 U_2} \approx 9$
Probing fragmented condensates

Shaking experiments

c-w/0307402, D.W. Wang, E.D., M. Lukin

D - displacement of the confining potential

Dipole mode

Chaos

Self-trapping

? Bose glass?

Power spectra

log P(w)

Dipole

log P(w)

chaos

log P(w)

Self-trapping
Summary

• Optical lattices are an efficient tool for reaching superfluidity of fermionic atoms

• Probes of superfluidity: second order interference, collective mode

• Two-component boson mixtures can be used to design spin-$\frac{1}{2}$ quantum systems

• Spin 1 bosons in optical lattices have a rich phase diagram with several insulating and superfluid phases

• Strongly fragmented quasi one-dimensional condensates in magnetic microtraps can be probed by shaking experiments. Several types of dynamic response are possible. It may be possible to study the localized regime.