Dynamics and pinning of vortices in a rotating BEC

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Outline

- Introduction to vortices and Bose-Einstein condensation (BEC)
- Dynamics of a single vortex line in a BEC, at zero and at finite temperature
- (Pinning of) vortex lattices in a BEC, structural transitions
- Conclusions
Vortices in classical fluids (I)

- Examples: hurricanes, tornadoes, water draining from bath-tub
Vortices in classical fluids (II)

- Classical fluid: described by density \( \rho \) and velocity-field \( \mathbf{v} \).
- Vorticity field: \( \mathbf{\omega} = \nabla \times \mathbf{v} \)
- Circulation around contour \( C \):
  \[
  \Gamma = \oint_C d\mathbf{l} \cdot \mathbf{v}
  \]
- Circulation constant of motion (Kelvin’s circulation theorem), can have any value
- Vortices observable by reduction of density
Vortices in a BEC: Experiment

http://jilawww.colorado.edu/bec, group of E.A. Cornell
Vortices in superfluids

- Superfluid (also superconductor) is described by macroscopic wave function (superfluid order parameter), complex-valued field:
  \[ \Psi = \sqrt{\rho} e^{i\theta} \]

- Superfluid velocity:
  \[ \vec{v} = \frac{\hbar}{m} \vec{\nabla} \theta \]

Wave function single-valued: circulation is quantized, quantized vortices

Multiply-quantized vortex energetically unstable
BEC: Experimental Setup

- Load laser-cooled atoms in harmonic magnetic trap
- Remove atoms with high energies using evaporative cooling
- Relax to equilibrium
- \( \rightarrow \) BEC!
Atomic BEC: First realization

Anderson et al. (1995), group of 2001 Nobel laureates E. A. Cornell and C.E. Wieman
Vortices in a BEC (I)

- At $T=0$ the condensate is well described by the Gross-Pitaevskii equation:

$$i\hbar \frac{\partial \Psi(\vec{x},t)}{\partial t} = \left( -\frac{\hbar^2 \nabla^2}{2m} + V_{\text{ext}}(\vec{x}) + \frac{4\pi a \hbar^2}{m} |\Psi(\vec{x},t)|^2 \right) \Psi(\vec{x},t)$$

- We have $\Psi = \sqrt{\rho} e^{i\theta}$ with $\vec{v}(\vec{x},t) = \frac{\hbar}{m} \nabla \theta(\vec{x},t)$

the superfluid velocity which is irrotational. Vortices form if the condensate is stirred.
Vortices in a BEC (II)

- Wave function of a condensate with a vortex line at the center:
  \[ \Psi \propto e^{in\phi} \times "\text{density profile}" \]

- Single-valued condensate w.f.: \( n \) integer
- Multiply-quantized vortex decays because \((2n)^2 > n^2 + n^2\), like vortices repel
- Superflow everywhere irrotational except for the origin
  - Condensate density vanishes at position of vortex. This is experimentally observable
Vortices: Experiments (I)

- Stirring the condensate:
  - Use laser as a spoon [Dalibard’s group, Ketterle’s group]
  - Phase imprinting method [Cornell + Wieman, Ketterle]
  - Condense from rotating thermal cloud [Cornell’s group]

- Above a certain critical frequency vortices are observed [theoretical work, Dalfovo, Stringari, …]

- Faster rotation: Vortex lattice

Vortices: Experiments (II)

Abrikosov lattice
Vortices: Experiments (III)

http://cua.mit.edu/ketterle_group/Nice_pics.htm
Vortex Precession: Experiment

- Nucleate one vortex by rotating the condensate, then stop rotating
- Vortex spirals outwards
- How to understand this theoretically?

http://jilawww.colorado.edu/bec, group of E.A. Cornell
Vortex dynamics at zero T (I)

- For simplicity: pancake-shaped condensate (curvature of vortex line not taken into account, \( q_z \ll q \))
- Variational approach to get physical insight (exact for noninteracting case, with vortex at origin):

\[
\Psi \propto \left\{ x - u_x(t) \right\} + i \left\{ y - u_y(t) \right\} \times \exp \left\{ -\frac{x^2 + y^2}{2q^2} - \frac{z^2}{2q_z^2} \right\}
\]

\[\propto e^{i\phi}\]

condensate density

\( q \)

\( x, y \)
Using the Gross-Pitaevskii equation we get (for small $u$’s):

\[
\frac{du_x(t)}{dt} = -\omega_P u_y(t) \quad \text{and} \quad \frac{du_y(t)}{dt} = \omega_P u_x(t)
\]

- Vortex precesses with frequency $\omega_P$ that depends on the number of atoms and the interactions (scattering length $a$).

  [For a treatment of large number of atoms see Fetter and Svidzinsky, J. Phys.: Condens. Matter 13, R135 (2001), and references therein]

- Problem 1: if vortex is in the center at $t=0$ it remains there forever, contrary to experimental observation

- Problem 2: distance of the vortex from the center is conserved in time, again in disagreement with experiment

- Solution: go to nonzero temperature
Nonzero T: Noisy Dynamics

- At nonzero temperature $T$, the condensate is in contact with a heat bath, i.e., the thermal cloud.

\[ i\hbar \frac{\partial \Psi(\vec{x},t)}{\partial t} = \left[ -\frac{\hbar^2 \nabla^2}{2m} + V^{\text{ext}}(\vec{x}) - \mu - iR(\vec{x},t) + \frac{4\pi a\hbar^2}{m} |\Psi(\vec{x},t)|^2 \right] \Psi(\vec{x},t) + \eta(\vec{x},t) \]

Dissipation: Condensate growth or evaporation from thermal cloud

Fluctuations: Thermal fluctuations due to incoherent collisions

Fluctuation-dissipation theorem

- NB: No free parameters!!
Vortex Dynamics at nonzero T

- Generalization of the variational calculus yields:

\[
\frac{du_x(t)}{dt} = -\omega P u_y(t) + \gamma u_x(t) + \eta_x(t)
\]

\[
\frac{du_y(t)}{dt} = \omega P u_x(t) + \gamma u_y(t) + \eta_y(t)
\]

- Thermal fluctuations “kick” vortex out of the center of the condensate:

\[
\langle \eta_i(t) \eta_j(t') \rangle = \frac{12 m q^2 a^2 (k_B T)^2}{N_c \hbar^3} \delta (t - t') \delta_{ij} \equiv \sigma \delta (t - t') \delta_{ij}
\]

- Damping rate:

\[
\gamma = \frac{6}{\pi} \frac{\left( \frac{a}{q} \right)^2 k_B T}{\hbar}
\]
Vortex lifetime (I)

- Calculate average distance of vortex from center of condensate: 
  \[ r(t) = \left\langle \sqrt{u_x^2 + u_y^2} \right\rangle(t) \]

- Equation for \( r(t) \):
  \[ \frac{dr(t)}{dt} = \gamma r(t) + \frac{\sigma}{2r(t)} \]

  \[ r(t) = \sqrt{\frac{2\gamma r^2(0) + \sigma}{2\gamma}} e^{2\gamma t} - \sigma } \]
Vortex Lifetime (II)

- Calculate average time it takes the vortex to go from $r_{min}$ to $r_{max}$:

$$\tau = \frac{1}{2\gamma} \log \left[ \frac{r_{max}^2 + \sigma}{2\gamma} \right] + \frac{1}{2\gamma} \log \left[ \frac{r_{min}^2 + \sigma}{2\gamma} \right]$$

- In the absence of thermal fluctuations, the vortex lifetime is infinite if initial position is the center of condensate (See also Fedichev and Shlyapnikov, 1999)

- Low temperature behaviour: $\tau \sim T^{-1} \log \left[ \frac{N_c \hbar \omega}{k_B T} \right]$
Vortex pinning in a BEC: Motivation

- Pinning of vortices due to intrinsic defects in the material active area of research in the field of superconductivity, and superfluid Helium.

- As a function of the strength of the (for simplicity) periodic pinning potential, the system exhibits transitions between various phases of the vortex lattice.

- Our proposal: BEC in a co-rotating optical lattice is ideal experimental system to study these transitions.

- Rotating optical lattice may be experimentally realized by rotating phase plates.
Variational ansatz (I)

- GP equation in rotating frame: (zero $T$, ground-state)
  \[ i\hbar \frac{\partial \Psi(\vec{x},t)}{\partial t} = \frac{\delta}{\delta \Psi^*(\vec{x},t)} H[\Psi^*, \Psi] \]

\[
H[\Psi^*, \Psi] = \int d\vec{x} \Psi^*(\vec{x},t) \left[-\frac{\hbar^2 \nabla^2}{2m} + V_{OL}(\vec{x}) + \frac{1}{2} \frac{4\pi a \hbar^2}{m} |\Psi(\vec{x},t)|^2 \right] \Psi(\vec{x},t)
\]

- Optical lattice potential:
  \[ V_{OL}(\vec{x}) = V_0 \left[ \sin^2(qx) + \sin^2(qy) \right] \]

- Assumptions: ignore trapping potential (infinite extent in $x$-$y$ plane. Tightly confined in $z$-direction (effective thickness $d$)

- Assume $q\xi << 1$, $V_0 << \mu$. TF limit, global density profile, w/o vortices:
  \[ n_{TF}(\vec{x}) = |\Psi(x)|^2 = \frac{m}{4\pi a \hbar^2} \left[ \mu - V_{OL}(\vec{x}) \right] \]
Variational ansatz (II)

- Ansatz for condensate wave-function, containing one vortex:
  \[
  \Psi_{\bar{u}}(\bar{x}) = \sqrt{n_{TF}(\bar{x})} \Theta \left( \frac{|\bar{x} - \bar{u}|}{\xi - 1} \right) \exp(i\phi)
  \]

- Generalization to \( N \) vortices straightforward, provided the vortex density is low:
  \[
  \Psi_N = \left[ \frac{1}{\sqrt{n_{TF}}} \right]^{N-1} \prod_{j=1}^{N} \left[ \Psi_{u_j} \right]
  \]
Vortex pinning potential

- Potential energy of a single vortex in a condensate loaded in an optical lattice:

\[
U_{\text{pinning}} (\vec{u}) \equiv H \left[ \Psi^* \vec{u}, \Psi \vec{u} \right] = \frac{d_z V_0}{8a} Q(q\xi) \left[ \cos(2qu_x) + \cos(2qu_y) \right]
\]

\[
Q(z) = J_1(2z)/(2z) + \int_{\rho=1}^{\infty} d\rho J_0(2z\rho)/\rho
\]

- Vortices are ‘pinned’ to maxima of optical potential.
Vortex-vortex interactions

- Energy of two vortices (subtracting contribution of optical lattice)

\[ V(|\vec{u}_1 - \vec{u}_2|) = H \left[ \Psi^*_{\vec{u}_1,\vec{u}_2}, \Psi_{\vec{u}_1,\vec{u}_2} \right] = -\frac{2\pi d_z \hbar^2 n}{m} \log \left( \frac{|\vec{u}_1 - \vec{u}_2|}{\xi} \right) \]

- Well-known logarithmic interaction potential (2D QED).

- Low vortex density: take only two-body interactions into account
Energy of vortex lattice

- We assume that the vortices form a lattice.
- For a given vortex density, and shape of the unit cell of the lattice, we are able to calculate the total energy of the vortex lattice [See: L.J. Campbell \textit{et. al.}, Phys. Rev. A \textbf{39}, 5436 (1989)]
- Find minimum energy configuration for a given strength of optical potential
Phase diagram – 2D optical lattice

- One vortex per pinning center

![Graph showing different lattice structures](image-url)

- Abrikosov lattice (triangular)
- Square-pinned lattice
- Half-pinned lattice

Graph coordinates: $V_0/\mu$ vs. $\xi q$
Phase diagram – 1D optical lattice

Pinned triangular lattice

\( V_0 / \mu \)

\( q_0 \equiv 2\pi \sqrt{3} / (3L) \)

\( q \xi = 0.01 \)

Abrikosov lattice (triangular)
Numerical solution of GP equation
Conclusions and outlook

- BEC in a co-rotating optical lattice experimentally attractive system to study vortex pinning at zero $T$

- Work in progress: collective modes of pinned lattices, acquire a gap

- Single vortex dynamics: decay of vortex understood as finite $T$ effect