To Carry Momentum or Not to Carry Momentum?

Two Meanings of Normal Fluid Velocity, and their Relevance to Supersolids

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Two Types of Superfluidity have been proposed

- Andreev and Lifshitz thought in terms of extremely mobile vacancies subject to quantum hopping and well-defined quantum states. Bose-Einstein Condensation of vacancy states (like holes?) would lead to superfluidity.

- Leggett thought in terms of Non-Classical Rotational Inertia (NCRI), where in the frame of rotating walls the ground state energy of a solid would be sensitive to the rotation rate.
Searches for Superfluidity

- Unsuccessful searches for flow of macroscopic object through solid 4He, or for flow of solid 4He under excess pressure, as suggested by Andrev and Lifshitz. Mezhov-Deglin, Greywall, Beamish.

- Successful search for NCRI by Kim and Chan, for solid 4He in Vycor, porous gold, and in bulk; and recently in solid para-H2.
A Problem for Theory

- Expect two fluids: normal and superfluid, with densities $\rho_n$ and $\rho_s$ that sum to the total average density $\rho$.

- At $T=0$ expect no normal fluid density, since it is attributed to excitations, which don’t occur at $T=0$.

- Measured superfluid fraction $f_s$ is only 1-2% at low $T$.

- Where is the rest of the density?
An Analogy

- Like a superfluid in a fine powder or connected network, include a velocity and density for the powder or network.

- Now have three velocities and three densities! But expect that the normal fluid and the powder velocities will lock together. Still, can’t assume this. Should derive it.
Very Complex: Let’s Back Up

- What about solid 4He for $T > 200$ mK, where there is no observed NCRI?

- Seems to support second sound, as for superfluid 4He. Have two velocities in superfluid 4He; same in solid 4He.

- Is second sound the same in both cases?
Liquid 4He

- Variables: density $\rho$, entropy density $s$, momentum density $g$, superfluid velocity $v_s$.

- Both velocities are true Galilean velocities, increasing by $v$ if the system is boosted by $v$. $g$ depends on both:
  $$g = \rho_n v_n + \rho_s v_s = \rho v_n + \rho_s(v_s - v_n)$$

- Equations for $\rho$, $s$, $g$, and $v_s$ give sound(s): first (mostly $\rho$ and non-zero $g$); density wave. second (mostly $s$ and $g=0$); temperature wave.
Solid 4He (T>200 mK)

- Suggested by Peshkov in 1946 (he observed second sound in liquid 4He in 1944). Sound in the gas of excitations, due to collisions between the excitations.

- Ward and Wilks in 1952 gave kinetic theory of phonon distribution, including energy and momentum moments. Only momentum (quasi-momentum) conserving collisions considered; no Umklapp or isotope scattering. And no macroscopic interpretation.
More

- Sussman and Thellung, and Gurzhi, in the early 1960’s, both studied second sound and heat flow for the phonon gas without Umklapps, which is slowed down only by surface scattering.

- They studied the phonon distribution and its energy and momentum moments in terms of the temperature and drift velocity of phonons.

- Mezhov-Deglin observed the predicted Poisueille flow of heat in the mid-60’s. (Unusual dependence on sample radius, for cylinder.)
Conditions for Second Sound


- Studied conditions for second sound in solid 4He.
  \[ \tau_U^{-1} << \omega << \tau_N^{-1} \]
  (Umklapp processes slow. Normal processes fast.)

- Second sound observed by Ackerman et al in 1966 in solid 4He above 200 mK. T=?
Late 60’s, early 70’s

- More theories of second sound in solids, attempts to merge hydrodynamics and kinetic theory.

Andreev and Lifshitz


- In principle have three velocities: normal fluid, superfluid, lattice. Only two carry momentum (normal, superfluid). Umklapp events lock normal fluid to lattice.

- Get two modes, one like elastic waves, one like 2nd sound (really, more like 4th sound).
Spirits of “Hydrodynamics”

- Method of “hydrodynamics” (Landau, Onsager, Khalatnikov) applies to long-wavelength, low-frequency situations.

1. Write down the thermodynamics in per unit volume form.

2. Write down “conservation laws” for thermodynamics densities, with unknown flux terms.

3. Solve for non-negative rate of entropy production in terms of products of fluxes and thermodynamics forces (like temperature gradient).

4. Deduce form of the unknown fluxes, consistent with space-time transformation properties.
Fleming and Cohen


- Two velocities: normal fluid and lattice, and only normal fluid carries momentum, but Umklapp processes lock them together.

- Vacancies mean that tracking lattice density doesn’t mean tracking mass density.

- Vacancies diffuse (new mode), as does heat.
Saslow (1977) and Liu (1978)


- Liu: Showed that, for internal consistency, Saslow’s superfluid velocity should have carried momentum, and when that was done, the theory reduced to A&L.

- Following a remark of Halperin and Hohenberg about a possible (but not needed) heat flux term (proportional to $v_n - v_s$) in the theory of superfluid hydrodynamics, Liu developed a hydrodynamics where the superfluid velocity consistently was not associated with momentum; equivalent to previous macroscopic theories of Enz and others.
A&L’s theory leads to ordinary elastic waves that couple to the superfluid, and a mode that is related to fourth sound.

A&L’s theory leads to transverse elastic waves that don’t couple to the superfluid.
Moving Forward

- Need to generalize momentum density to include another velocity, to \( g = \rho_n v_n + \rho_s v_s + \rho_R v_R \), where \( R \) means the Rest of the lattice.

- Do not equate \( v_R \) to lattice velocity \( v_L = du/dt \); let the theory determine that.

- Work is in progress. Harry Kojima (Rutgers) is planning experiments.
\[ d\varepsilon = T ds + \lambda_{ik} dw_{ik} + \mu d\rho + \nu_{ni} dg_i + j_{R_i} dv_{R_i} \]
\[ \varepsilon = -P + Ts + \lambda_{ik} w_{ik} + \mu \rho + \nu_{ni} g_i + j_{R_i} v_{R_i} \]

\[ \partial_t \varepsilon + \partial_i Q_i = 0, \]
\[ \partial_t \rho + \partial_i g_i = 0, \]
\[ \partial_t g_i + \partial_k \Pi_{ik} = 0, \]
\[ \partial_t s + \partial_i f_i = \frac{R}{T}, \quad (R \geq 0), \]
\[ \partial_t u_i + Y_i = 0, \]
\[ \partial_t v_{R_i} + Z_i = 0. \]
Now determine unknown fluxes, like $f_i$. 

\[ R = -\partial_i\left[ Q_i - T f_i - \mu g_i - \lambda_{ik}(Y_k + v_{nk}) - v_{nk}\Pi_{ki} + P v_{ni} - v_{ni}w_{jk}\lambda_{jk} + v_{nj}w_{jk}\lambda_{ik} + v_{ni}v_{Rk}j_{Rk}\right] \]

\[-(f_i - sv_{ni})\partial_i T - (\partial_i\mu - Z_k + v_{ni}\partial_i v_{Rk})j_{Ri} - (Y_k + v_{nk} - v_{nj}w_{jk})\partial_i\lambda_{ik} \]

\[+(P\delta_{ik} + v_{nk}g_{i} + \lambda_{kj}w_{ij} - \lambda_{jl}w_{jl}\delta_{ik} - j_{Rj}v_{Rj}\delta_{ik} - \lambda_{ki} - \Pi_{ik})\partial_k v_{ni}. \]
Some New Results

- Can develop a theory where $v_R$ is driven by the negative gradient of the chemical potential and is subject to lattice drag.

- At low frequencies, the drag wins. At high frequencies, the negative gradient of the chemical potential wins.

- Need experimental input.