2D weak-localization in the presence of Elliot-Yafet spin-relaxation

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Outline

- Weak-localization and anomalous magnetoconductivity
- Method of conductivity calculation
- Weak-localization in diffusive and ballistic modes with account of Elliot-Yafet spin relaxation
Introduction

Weak-localization and anomalous magnetoconductivity
2D conductivity

Magnetic field is perpendicular to the plane of the system.
Classical magnetoconductivity

\[ \sigma_{xx} = \frac{ne^2\tau}{m(1+(\omega_c\tau)^2)} \]

\[ \sigma_{xy} = \omega_c\tau \sigma_{xx} \]

At low fields
\[ \omega_c\tau \ll 1 \]

\[ \sigma_{xx}(B) \approx \frac{ne^2\tau}{m} \]

\[ \omega_c = \frac{eB}{m} \]
Evidence of weak localization

Magnetococonductivity of thin Cu film ($\sigma_{xx}$)


Magnetoresistance of thin Cu film ($\rho_{xx}$), $\omega_c\tau < 10^{-4}$

Bergmann (1984)
Theoretical explanation

\[ k_F l \gg 1 \] - many wave function oscillations between scatterings

Paths with closed regions decrease conductivity
Closed paths

The phase shift along each of two paths

\[ \Delta \varphi = k_F L \]

The main conductivity term

\[ \frac{e^2}{2\pi \hbar} k_F l \]

The weak localization conductivity term

\[ \frac{e^2}{2\pi \hbar} \ln \left( \frac{l_\varphi}{l} \right) \]

\( l_\varphi \) – maximal length of the loop without destruction of coherence
Influence of magnetic field

The phase shift due to the magnetic field ($A$ – vector potential)

$$\Delta \varphi = 2\frac{e}{c} \int A dL$$

or

$$\Delta \varphi = 2\frac{e}{c} \int H dS$$

The magnetic field decreases interference.
Diffusive and ballistic modes

- Diffusive mode – trajectories with large number of scatters interfere.

\[ \frac{\hbar k_F \tau}{m} = l \ll l_H = \sqrt{\frac{\hbar c}{eH}} \] - weak magnetic field.

- Ballistic mode – only trajectories with small number of scatters interfere. In sufficiently strong magnetic fields.
Quantum correction (diffusive mode)

Hikami, Larkin, Nagaoka (1979)

\[ l \ll l_H \]

**Asymptotic:**

H is small

\[ \Delta \sigma(H) = const \cdot H^2 \]

H is large

\[ \Delta \sigma(H) = const \cdot \ln H \]
Quantum correction (diffusive and ballistic mode)

Asymptotic:

H is small
\[ \Delta \sigma(H) = \text{const} \cdot H^2 \]

H is large
\[ \Delta \sigma(H) = \Delta \sigma(\infty) - \frac{\text{const}}{\sqrt{H}} \]

Kawabata (1984)
Spin relaxation

- Dyakonov-Perel – carriers spin rotates between scatterings (semiconductors with wide band gap)

- Elliot-Yafet – spin dependent scatterings (semiconductors with narrow band gap)
Spin relaxation and weak localization

Magnetoresistance of InGaAs/InP quantum wells at different temperatures. Polyanskaya et al. (2002)

Theoretical curve for diffusive mode, considering spin HLN 1979
The main objective of the current work:

- Calculation of weak localization correction to magnetoconductivity with account of Elliot-Yafet spin relaxation in both diffusive and ballistic mode.
Zero-temperature Green function method is used for calculation of anomalous magnetoconductivity
Diagrams

Classical conductivity

Weak localization correction

Correlator

\[ W(k \to k', q \to q') = \sum_{a,b} V_a(k \to k')V_b(q \to q') \]

Green function

\[ G(k, \varepsilon) = \frac{1}{\varepsilon - \frac{p^2}{2m} + \frac{isign(\varepsilon)}{2\tau}} \]
\[ \int G(k, \varepsilon)G(k, \varepsilon - \omega)k^2\,dk \]
Weak localization correction

or

where

= + + ...
Cooperon equation

Analytical form:

\[ C(k, k', Q) = W + \int W G^A(-g) G^R(Q + g) C(g, k', Q) dg \]

Diffusive mode: Q is small. So kernel can be expressed as series over Q.
Cooperon without spin

Diffusive approximation

\[ C(k,k',Q) = \frac{W}{DQ^2\tau} \]

Exact solution (Dyakonov (1994))

\[ C(k,k',Q) = \frac{W}{1 - \frac{1}{\sqrt{1 + \frac{\tau}{\tau_\phi} + DQ^2\tau}}} \left( \frac{1}{\sqrt{1 + \frac{\tau}{\tau_\phi} + DQ^2\tau}} \right)^2 \]
Spin-orbit interaction

Matrix elements

\[ V_{\alpha\beta}(k \rightarrow k', r) = V_0 \left( \delta_{\alpha\beta} F_0(k, k', r) + F_1(k, k', r) \left[ \sigma_{\alpha\beta} \times (k + k') \right]_z \right) \]

Correlation of pair scattering

\[ W_{\alpha\beta \gamma \kappa}(k \rightarrow k', q \rightarrow q') = W \left( \delta_{\alpha\beta} \delta_{\gamma \kappa} + c_x \left[ \sigma_{\alpha\beta} \times (k + k') \right]_z \left[ \sigma_{\gamma \kappa} \times (q + q') \right]_z \right) \]

\[ 2c_x k_F^2 = \frac{\tau}{\tau_s} \]
Main problem of Cooperon calculation

\[ W_{\alpha\beta\gamma\kappa}(k \rightarrow k', q \rightarrow q') = W \left[ \delta_{\alpha\beta} \delta_{\gamma\kappa} + c_x \left[ \sigma_{\alpha\beta} \times (k + k') \right]_z \left[ \sigma_{\gamma\kappa} \times (q + q') \right]_z \right] \]

\[ C(k, k', Q) = W (-k \rightarrow -k', Q + k \rightarrow Q + k') + \int W (-k \rightarrow -g, Q + k \rightarrow Q + g) G^A(g) G^R (g - Q) C(g, k', Q) dg \]

Angular harmonics of \( k, k' \) and \( Q \) are present in ballistic limit.

In the diffusive mode \( Q \) is small and angular dependencies can be averaged.
Solution of cooperon equation in the presence on Elliot-Yafet spin relaxation

\[ W_{\alpha\beta\gamma\kappa}(k \rightarrow k', q \rightarrow q') = W \left[ \delta_{\alpha\beta} \delta_{\gamma\kappa} + c_x \left[ \sigma_{\alpha\beta} \times (k + k') \right]_z \left[ \sigma_{\gamma\kappa} \times (q + q') \right]_z \right] \]

\( W \) contain 4 harmonics over \( k \) and \( k' \)

\[ C(k, k', Q) = W (-k \rightarrow -k', Q + k \rightarrow Q + k') + \]

\[ \int W(-k \rightarrow -g, Q + k \rightarrow Q + g)G^A(g)G^R(g - Q)C(g, k', Q)dg \]

Expansion of \( C \) over harmonics of \( k \).
\( C \) also contains 4 harmonics over \( k \) => number of harmonics is finite
Conductivity corrections without magnetic field

Without spin (Dmitriev et al. 1997):

$$
\Delta \sigma = -\frac{e^2}{2\pi^2 h} \left[ \ln \left(1 + \frac{\tau}{\tau_\phi} \right) + \frac{\ln \left(1 + \frac{\tau_\phi}{\tau} \right)}{1 + 2 \frac{\tau_\phi}{\tau}} - \frac{\ln 2}{1 + \tau / 2\tau_\phi} \right]
$$

With spin:

$$
\Delta \sigma = -\frac{e^2}{2\pi^2 h} \left[ \ln \left(1 + \frac{\tau_\phi}{\tau} \right) \left(1 + \frac{\tau}{2\tau_\phi} + \frac{11}{2} \frac{\tau}{\tau_s} \right) - \frac{1}{2} \ln \left(1 + \frac{2\tau_\phi}{\tau_s} \right) - \ln 2 \right]
$$

$\tau_\phi$ – phase relaxation time, $\tau$ – momentum relaxation time, $\tau_s$ – spin relaxation time
Magnetoconductivity

Asymptotic at large magnetic fields

\[ \sigma(H) - \sigma(\infty) = \text{const} \left(1 - 3 \frac{\tau}{\tau_s}\right) \frac{1}{\sqrt{H}} \]

\[ \frac{\tau}{\tau_\varphi} = 0.01 \]
Magnetoconductivity

S. Pedersen, et al.
($l_b = l$ at 60 Gauss)
Conclusions

- The weak localization theory with the account of Elliot-Yafet spin relaxation in both diffusive and ballistic modes was developed.
- The dependence of anomalous magnetoconductivity on the magnetic field and material parameters was calculated.