Optical Conductivity of 2D Electron Systems

Eugene Mishchenko

University of Utah


A. Farid and E.M., cond-mat/0603058

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Outline

-- Quasiparticles and collective modes (plasmons) in Fermi Liquid

-- Absorption, optical conductivity, Landau damping -- RPA

-- Plasmon attenuation: beyond RPA

-- 2D system with spin-orbit coupling: interplay of spin-orbit coupling and many-body effects
Fermi Liquid

\[ \xi_p = \frac{p^2}{2m} - \frac{p_F^2}{2m} \]

\[ \psi_p = e^{-i\xi_p t - \Gamma t} \]

Quasiparticles are well defined: \[ \Gamma \equiv \xi_p \]

\[ \Gamma_p = 2\pi \int d\vec{k} d\vec{q} |U|^2 n_k (1 - n_{p+q})(1 - n_{k-q}) \]

\[ \Gamma_p = \begin{cases} \xi_p^2 / E_F, & 3D \\ (\xi_p^2 / E_F) \ln(E_F / \xi_p), & 2D \end{cases} \]

Quinn & Ferrel (1958)

Chaplik (1972), Zheng & Das Sarma (1996)

Jungwirth & MacDonald (1996)
Collective modes (plasmons)

Hydrodynamic approach (equivalent to RPA)

| Continuity equation: | $\omega \rho = \vec{q} \cdot \vec{j}$ |
| Equation of motion: | $m \omega \vec{j} / e = e \vec{q} \varphi$ |
| Poisson equation: | $e \varphi = V_q \delta \rho$ |

$\omega_q^2 = \frac{nq^2}{m} V_q$

Short-range interaction: $V_q = V \quad \rightarrow \quad \omega_q \propto q$ \quad zero sound

Unscreened interaction:

| 3D | $V_q \propto q^{-2}$ | $\omega_q = \text{const}$ |
| 2D | $V_q \propto q^{-1}$ | $\omega_q \propto \sqrt{q}$ |
| 1D | $V_q \propto \ln q$ | $\omega_q = q\sqrt{\ln q}$ |
Absorption in Fermi Liquid

Single-particle channel: Landau Damping

\[ \omega = \xi_{p+q} - \xi_p \approx \vec{q} \cdot \vec{v} < qv_F \]

No Landau Damping

Plasmons do not decay via the single-particle channel

Go beyond RPA!
Beyond RPA: two-particle channel

Landau damping:
\[ \omega = \vec{q} \cdot \vec{v} < qv_F \]

Two-pair channel:
Two pairs moving in opposite direction can have large energy while having negligible total momentum
Beyond RPA: two-particle channel

\[ \Pi = \text{RPA} \]

\[ \Pi = \text{screened RPA interaction} \]

3D: DuBois & Kivelson, 1969
Our method: many-body transitions in the presence of external field

\[ \phi(\vec{x}, t) = \phi_0 e^{-i\omega t + i\vec{q} \cdot \vec{x}} + \phi_0^* e^{i\omega t - i\vec{q} \cdot \vec{x}} \]

\[ -\frac{dW}{dt} = \text{absorption} - \text{emission} \]

\[ -\frac{dW}{dt} = 2q^2 |\phi_0|^2 \sigma'(\omega, q) \]

real part of the optical conductivity

all real transitions
Relation between plasmon attenuation and optical conductivity

optical conductivity $\sigma'(\omega, q)$ measures absorption at frequency $\omega$ and wavevector $q$

plasmon attenuation $\omega \rightarrow \omega_q$

$$\Gamma_q = \frac{q^2 V_q}{2e^2} \sigma'(\omega_q, q)$$

$$\omega_q^2 = \frac{nq^2}{m} V_q$$

$$\Pi(\omega, q) = -i \frac{q^2}{\omega e^2} \sigma(\omega, q)$$

Ward identity
Formalism: Golden Rule

Two-particle wave function

\[ \Psi_{pk} = \frac{1}{\sqrt{2}} \left( \psi_p(x_1)\psi_k(x_2) - \psi_p(x_2)\psi_k(x_1) \right) \]

Need transition probabilities:

\[ \Psi_{pk} \rightarrow \Psi_{p'k'} \]

Second-order time-dependent perturbation theory in \( H_1 \)

First-order Hamiltonian:

\[ H_0 = -\frac{\hbar^2 \nabla^2_1}{2m} - \frac{\hbar^2 \nabla^2_2}{2m} \]

First-order corrected Hamiltonian:

\[ H_1 = e\phi(\vec{x}_1) + e\phi(\vec{x}_2) + V(\vec{x}_1 - \vec{x}_2) \]
Transition amplitudes

\[
W_{p_k \to p'_k'} = \frac{2\pi}{\hbar} |M|^2 \delta(\xi_p + \xi_k - \xi_{p'} - \xi_{k'} + \hbar \omega) \delta(\bar{p} + \bar{k} - \bar{p}' - \bar{k}' + \hbar \bar{q})
\]

\[
M = \begin{array}{c}
p \\
k \\
p' \\
k'
\end{array} + \begin{array}{c}
p \\
k \\
p' \\
k'
\end{array}
\]

\[
M = \begin{array}{c}
p \\
k \\
p' \\
k'
\end{array} + \begin{array}{c}
p \\
k \\
p' \\
k'
\end{array} + \text{exchange graphs}
\]

\[
e\phi_0 \frac{V_{p'-p} \pm V_{k'-p}}{\xi_k - \xi_{k+q} + \omega}
\]

How to read graphs:

\[
= e\phi_0 \\
= V_{p'-p}
\]

virtual = \((E_i - E_V)^{-1}\)
Optical conductivity

\[
\sigma'(\omega, q) = \frac{1}{2q^2 |\phi_0|^2} \frac{-dQ}{dt}
\]

\[
\frac{-dQ}{dt} = \frac{-dW_{\text{abs}}}{dt} + \frac{dW_{\text{em}}}{dt}
\]

\[
\frac{dW_{\text{em}}}{dt} = \frac{dW_{\text{abs}}}{dt} \exp(-\omega/T)
\]

Detailed balance principle

\[
M = \begin{array}{c}
\begin{array}{c}
\begin{array}{c}
p \;\;\;\;\;\;\;\;\;\;\;\; p' \\
\downarrow \\
k \\
\downarrow \\
k'
\end{array}
\end{array} + \begin{array}{c}
\begin{array}{c}
p \;\;\;\;\;\;\;\;\;\;\;\; p' \\
\downarrow \\
k \\
\downarrow \\
k'
\end{array}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
p \;\;\;\;\;\;\;\;\;\;\;\; p' \\
\downarrow \\
p-p' \\
\downarrow \\
k \\
\downarrow \\
k'
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
p \;\;\;\;\;\;\;\;\;\;\;\; p' \\
\downarrow \\
k \\
\downarrow \\
k'
\end{array}
\end{array} + \begin{array}{c}
\begin{array}{c}
p \;\;\;\;\;\;\;\;\;\;\;\; p' \\
\downarrow \\
k \\
\downarrow \\
k'
\end{array}
\end{array}
\\end{array}
\]

Each matrix element is not small in \( q \)

but they \textit{twice} interfere \textit{pairwise} almost canceling each other leading to \( M \ll q^2 \)

Optical conductivity vanishes in the homogeneous limit \( q \rightarrow 0 \)

as \( \sigma(\omega, q) \ll q^2 \)
Plasmon attenuation

\[ \sigma'(\omega, q) = \frac{e^2 q^2}{12 \pi^2 \hbar^2 \omega^2} \left[ \omega^2 + (2\pi T)^2 \right] \ln \frac{E_F}{\omega} \]

\[ \Gamma_q = \frac{q^2 V_q}{2e^2} \sigma'(\omega_q, q) = \frac{q^2}{24 \pi \hbar^2 E F} \left[ \omega_q^2 + (2\pi T)^2 \right] \ln \frac{E_F}{\omega_q} \]

\[ \omega_q \propto \sqrt{q} \quad \rightarrow \quad \Gamma_q / \omega_q \propto q^{5/2} \]

Plasmon attenuation is independent of the electron charge: Screened interaction (unitary limit)
1. Discussion of optical conductivity

\[
\sigma'(\omega, q) = \frac{e^2 q^2}{12\pi^2 p_F^2 \omega^2} \left[ C_0^2 + (2\pi T)^2 \right] \ln \frac{E_F}{\omega}
\]

Logarithmic enhancement – trace of one-dimensional singularities in 1D

Exchange processes are dominated by direct processes by virtue of the logarithm
2. Discussion of optical conductivity

\[ \sigma'(\omega, q) = \frac{e^2 q^2}{12\pi^2 p_F^2 \omega^2} \left( \omega^2 + (2\pi T)^2 \right) \ln \frac{E_F}{\omega} \]

Optical conductivity vanishes in the homogeneous limit \( q \to 0 \)

- **Galilean invariance**
- **Parabolic spectrum** \( \epsilon(p) = \frac{p^2}{2m} \)

Electric current depends only on the total momentum and is not modified by electron-electron collisions in the homogeneous limit

\[ \vec{j} = e \sum \frac{\vec{p}}{m} \]

**Many-body effects have no effect on** \( \sigma(\omega, 0) \)
Spin-orbit coupling breaks Galilean invariance

Particle moving in electric field:
\[ \vec{E} = -e^{-1} \nabla U \]

In the reference frame moving with the electron velocity there is a magnetic field
\[ \vec{B} = \frac{\vec{v} \times \vec{E}}{c} \]

This magnetic field leads to the Zeeman energy which is momentum-dependent
\[ H_{so} = \frac{g}{2m^2c^2} \nabla U \cdot \vec{s} \times \vec{p} \]

Electric current is spin-dependent and is not conserved during electron-electron interactions
\[ \vec{j} = e \sum \vec{v} = e \sum \left( \frac{\vec{p}}{m} + \nabla_p H_{so} \right) \]

\[ \sum \frac{\vec{p}}{m} = \text{const} \quad \rightarrow \quad \vec{j} \neq \text{const} \]

Homogeneous optical conductivity can probe many-body effects
2D electron system

GaAs/AlGaAs heterojunction

dopants

confined electrons

Electric field

\[ H_{SO} = \frac{g}{2m^2c^2} \nabla U \cdot \vec{s} \times \vec{p} \]

\[ H_{SO} = \lambda (\vec{s} \times \vec{p})_z \]

\[ = \lambda (s_x p_y - s_y p_x) \]

‘Rashba’ S-O Hamiltonian

(Vas’ko ’79, Bychkov & Rashba ’84)
Electron eigenstates

Spin degeneracy is lifted by \( H_{SO} = \lambda (s_x p_y - s_y p_x) \)

Eigenvalues:
\[
\varepsilon(p) = \frac{p^2}{2m} + a\lambda p
\]

chirality \( a = 1, -1 \)

Eigenstates:
\[
\psi^a_p = \frac{1}{\sqrt{2}} \begin{pmatrix}
  e^{i\theta/2} \\
  ae^{-i\theta/2}
\end{pmatrix}
\]

Different Fermi momenta:
\[
p^a_F = p_F - am\lambda
\]
Modification of Landau damping

Direct transitions between subbands are possible:

1) ‘Conventional’ Landau damping

2) ‘Combined’ or ‘chiral’ resonance

Energy constraint

\[ \omega = \varepsilon_1(p) - \varepsilon_2(p) = 2\lambda p \]

Partition constraint

\[ p_F - m\lambda < p < p_F + m\lambda \]

\[ \sigma'((\omega) \text{ with } \frac{e^2}{16} \]

\[ 4m\lambda^2 \]
Two-particle channel with spin-orbit

\[ k - Q + q \rightarrow p + Q \]

Direct processes

\[ \omega \rightarrow q \]

Exchange processes
Our method applied

\[ W_{p k \rightarrow p' k'} = \frac{2\pi}{\hbar} |M|^2 \delta(\xi_p^a + \xi_k^b - \xi_{p'}^c - \xi_{k'}^d + \hbar \omega) \delta(\vec{p} + \vec{k} - \vec{p}' - \vec{k}' + \hbar \vec{q}) \]

\[ M = \begin{array}{ccc}
 p, a & \nabla & p', c \\
 p + q, f & & \\
 k, b & \nabla & k', d
\end{array} \]

\[ A_{p, p'}^{ab} = (\chi_{p'}^b )^\dagger \cdot \chi_p^a = \frac{1}{2} (e^{i(\theta_p - \theta_{p'})/2} + a b e^{-i(\theta_p - \theta_{p'})/2}) \]

Projection of spin states before and after scattering

How to read graphs:

\[ A_{p, p'}^{ab} \] or \[ A_{p, p'}^{ab} \]
Optical conductivity with Spin-Orbit coupling

\[ M = \]

\[
\begin{array}{c}
\begin{array}{c}
p \quad p' \\
k \quad k'
\end{array}
\end{array}
+ \begin{array}{c}
\begin{array}{c}
p \quad p' \\
k \quad k
\end{array}
\end{array}
+ \begin{array}{c}
\begin{array}{c}
p' \\
k - k' + q
\end{array}
\end{array}
+ \begin{array}{c}
\begin{array}{c}
p' \\
k - k'
\end{array}
\end{array}
\]

The four terms interfere only once leading to \( M \square q \)

For a short-range interaction \( V = \text{const} \)

\[ \sigma'(\omega) = \frac{e^2 \lambda^2 m^2 V^2}{v_F^2 \omega^2} \left[ \omega^2 + (2\pi T)^2 \right] \]

This contribution is the result of the interplay of spin-orbit coupling and interaction
Absence of logarithmic factor – suppression of collinear scattering

No interplay of interactions and spin-orbit coupling in 1D

Chirality of colliding particles is conserved: No change in the total velocity (thus, current) during a collision in 1D
Exchange processes are equally important as the direct processes.
Discussion: III

\[ \sigma'(\omega) = \frac{e^2 \lambda^2 m^2 V^2}{v^2_f \omega^2} \left[ \omega^2 + (2\pi T)^2 \right] \]

Two-pair contribution is divergent for \( T \ll \omega \rightarrow 0 \)

Three- and higher-particle processes are important

\[ \text{Diagram} \]
Conclusions

Spin-orbit coupling results in a single-pair absorption

\[ \sigma_1'(\omega) \]

which is narrow in frequency

Combined effects of spin-orbit coupling and electron-electron interactions result in a broader contribution from many-particle excitations

Spin-orbit coupling makes optical conductivity a probe for many-body effects