The Neel to Bloch domain wall phase transition in ferromagnetic strips

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Statement of the problem

The transition from Neel domain wall to Bloch domain wall in ferromagnetic strips type I or type II phase transition?


Micromagnetical approach

Analytical approach

Type II phase transition
Outline

- Domain walls in ferromagnets
- Results of micromagnetical calculation
- Analytical description
- Comparison
Ferromagnetic Hamiltonian

The energy of the magnetization distribution in ferromagnet:

\[ W[\vec{M}(\vec{r})] = W_{\text{ex}}[\vec{M}(\vec{r})] + W_{d-d}[\vec{M}(\vec{r})] + W_{a}[\vec{M}(\vec{r})] \]

The exchange energy:

\[ W_{\text{ex}} = A \int \left( \nabla \cdot \vec{M}(\vec{r}) \right)^2 d\vec{r} \]

The dipole-dipole energy (the magnetic field energy):

\[ W_{d-d} = \iint \frac{1}{|\vec{r} - \vec{r}'|^3} \left( 3 \frac{(\vec{M}(\vec{r})(\vec{r} - \vec{r}'))(\vec{M}(\vec{r}')(\vec{r} - \vec{r}'))}{|\vec{r} - \vec{r}'|^2} - M(\vec{r})M(\vec{r}') \right) d\vec{r} d\vec{r}' \]

The anisotropy energy:

\[ W_{\text{ex}} = \int F(\vec{M}(\vec{r})) d\vec{r} \]
Domain wall

Large DD energy

Large exchange energy
Domain walls in strip

$L \gg w \gg h$

\[
\frac{l_{ex}^2}{hw} \ll 1
\]

\[
l_{ex} = \sqrt{\frac{A}{2 \pi M_s^2}}
\]
Domain wall types

- Neel domain wall
- Bloch domain wall
Limit of thin samples

\[ W_{d-d} = W_{d-d, \text{faces}} + W_{d-d, \text{sides}} \]

\[ W_{d-d, \text{faces}} \sim h \langle M_z^2 \rangle \]

\[ W_{d-d, \text{sides}} \sim h^2 \langle M_x^2 \rangle \]

Magnetization lies in plane x-y — Neel domain wall
Neel domain wall

Magnetization rotates around z-axis. Magnetization is uniform along y and z.

\[ \vec{M}(\vec{r}) = \vec{M}(x) \]

Bloch domain wall

Thick samples. Rotation around in-plane axis.

Phase transition

Small thickness
- Neel wall

Large thickness
- Bloch wall

Phase transition
Micromagnetical Simulation

\[ A = 1.3 \times 10^{-6} \text{ erg/cm} \]

\[ M_s = 795 \text{ emu/cm}^3 \]

\[ l_{ex} = 5.7\text{ nm} \]

\[ h = 1\text{ nm} - 50\text{ nm} \]

Calculated the lowest energy state with domain wall at different thicknesses of the strip.

Found the spectrum of oscillations and the lowest frequency oscillation.

K. Rivkin
Mode frequency goes to 0 at some thickness.

Micromagnetic simulation (K. Rivkin)

Mode — oscillations of the domain wall central part.
Micromagnetics results

- Mode performs shifting of Neel wall center along x
- The transition Neel wall to asymmetric Bloch wall occurs.

Cross-section y=const (K. Rivkin)
Symmetry of Neel wall

Magnetization distribution in cross-section $y=\text{const}$

Mirroring symmetry

\[ M_x(x, z) = M_x(\pm x, \pm z) \]
\[ M_y(x, z) = -M_y(-x, z) \]
\[ M_y(x, z) = M_y(x, z) \]

Micromagnetic simulation (K. Rivkin)
Symmetry of asymmetric Bloch wall

Central symmetry

\[ M_x(x, z) = M_x(-x, -z) \]
\[ M_y(x, z) = -M_y(-x, -z) \]
\[ M_z(x, z) = M_z(-x, -z) \]

Magnetization distribution in cross-section y=const

Micromagnetic simulation (K. Rivkin)
Transition mode symmetry

Neel wall + transition mode

= asymmetric Bloch wall

Mode symmetry - Mirroring symmetry

\[
M_x(x, z) = -M_x(-x, z) \\
M_x(x, z) = -M_x(x, -z) \\
M_y(x, z) = M_y(-x, z) \\
M_y(x, z) = -M_y(x, -z) \\
M_z(x, z) = M_z(\pm x, \pm z)
\]
Neel domain wall shape

Micromagnetical calculation in the limit of thin samples. (K. Rivkin)

\[ M_x(x) = \text{sech}\left(\frac{x}{\delta}\right) = \frac{1}{\cosh\left(\frac{x}{\delta}\right)} \]

\[ M_y(x) = \text{th}\left(\frac{x}{\delta}\right) \]

\[ M_x(x) = \Lambda \left( \cos\left(\frac{x}{w}\right) \text{ci}\left(\frac{x}{w}\right) + \sin\left(\frac{x}{w}\right) \text{si}\left(\frac{x}{w}\right) \right) \]

\[ M_y(x) = \text{sign}(x) \sqrt{1 - M_x(x)^2} \]

\( \delta \) — the half-width of domain wall center

Central region

Region of tails

K.Rivkin et. al., submitted
Mode magnetization

Neel domain wall magnetization:

\[ M^0(x) = (u(x), v(x), 0) \]

The mode should be orthogonal to the initial solution

\[ \mu(x, z) = (\lambda(x, z)v(x), -\lambda(x, z)u(x), \zeta(x, z)) \]

Mode shifts center of Neel domain wall:

\[ \lambda(x, z_0)v(x) \sim \frac{d}{dx}u(x) \]

\[ \lambda(x, z_0)u(x) \sim \frac{d}{dx}v(x) \]
Mode magnetization

\[
\mu(x, z) = \left( \lambda(x, z) v(x), -\lambda(x, z) u(x), \zeta(x, z) \right)
\]

Two z harmonics are taken

\[
\lambda = \text{sech} \left( \frac{x}{\delta} \right) \left( \lambda_1 \frac{2z}{h} + \lambda_3 \left( \frac{2z}{h} \right)^3 \right)
\]

\[
\zeta(x, z) = \text{sech} \left( \frac{x}{\delta} \right) \left[ 2 \text{sech} \left( \frac{x}{\delta} \right) - 1 \right] \left( \zeta_0 + \zeta_2 \left( \frac{2z}{h} \right)^2 \right)
\]
Critical condition

$$W_{[\lambda, \zeta]} = W_0 + \left( \lambda_1, \lambda_3, \zeta_0, \zeta_2 \right)^T M \left( \lambda_1, \lambda_3, \zeta_0, \zeta_2 \right) + \ldots$$

The second correction to the energy is 0.

$$W_{mode}[\lambda, \zeta] = \left( \lambda_1, \lambda_3, \zeta_0, \zeta_2 \right)^T M \left( \lambda_1, \lambda_3, \zeta_0, \zeta_2 \right)$$

Main contribution is from dipole-dipole energy.

Exchange energy is not sufficient at small exchange lengths

$$l_{ex} = \sqrt{\frac{A}{2 \pi M_s^2}}$$

Critical point condition

$$\det \left[ M \right] = 0$$
Critical thickness

\[ \delta \approx 0.4 h_c \]
Comparison

Transition points (micromagnetics)

Theoretical value

\[ \delta \approx 0.4 h_c \]

\[ h_c \approx 2.5 \delta \]

\( \delta \) — domain wall central part width, \( h \) — thickness of the strip
Free Energy

Transition mode vector:

\((\lambda_1, \lambda_3, \zeta_0, \zeta_2) = \eta (0.38, 0.7, 0.66, 0.27)\)

Energy expression

\(W_{\text{mode}}(\eta) = \eta^2 0.3 M_s^2 L h^2 \left( \frac{\delta}{h} - \frac{\delta}{h_c} \right) + 0.008 \eta^4 M_s^2 L h^2\)
Conclusions

- Analytical and micromagnetical description of Neel domain wall to asymmetric Bloch domain wall type II phase transition.
- The relation between critical thickness and Neel domain wall center width is obtained.