d-Wave superconductivity from loop currents in cuprates

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In cuprates Fermi surface quasiparticles coexist with local variables

**L-variables** in this talk
(aka “flux pair”, aka “circulating current”)

evidence

- fluctuations of L-variables couple to fermions and give rise to MFL behavior in the “strange metal” region of phase diagram

- L-variables order globally below $T = T^*(x)$: lattice symmetry is lowered and system acquires magnetic structure - TRSB.
outline of the talk

• (sketchy) survey of the phase diagram of cuprates and basic experimental facts
• survey of major experimental development in last ~3 years
• nature of local variables and motivation from mean field analysis
• Hamiltonian. classical and quantum fluctuations
• Coupling of L-variables to Fermi surface quasiparticles
• symmetry of superconducting pairing
“Strange metal” phase

Electrical resistivity
Thermal conductivity
Optical conductivity
Tunneling conductance
Hall coefficient
Nuclear relaxation rate

... are all anomalous.

MFL phenomenology
Varma et.al. (1989)
Resistivity, optical conductivity, Raman shift

Collins et.al. (1989) YBCO (Tc=92)


Gurevich, Fiory (1987)

Slakey et.al. (1991) YBCO

$B_{1g}$  $B_{2g}$  $A_{1g}$
“Pseudogap” phase

As we cross $T^*$ line,

gap appears on the Fermi surface

behavior of transport, thermodynamic properties changers -- crossover

no specific heat singularity

... thermodynamically sharp phase transition

Characteristic temperatures determined from crossover behavior

also ARPES, e.g., Timusk Stratt (1999)
Color-map indicates deviation of the resistivity from linear-in-T
Polarized neutron scattering.

- No new bragg peaks
- (subset of) Existing Bragg peaks acquire magnetic structure

\[(\text{spin-flip}) \text{ Neutron intensity } I(q) \propto |f(q)|^2 |S(q)|^2 |s(\perp)|^2\]

\[S(q) \propto \cos(\pi z L) \sin \left(2\pi x_0 (H \pm K)\right)\]

Neutron spin
\[s(\perp) = \hat{q} \times (s \times \hat{q})\]

\[q = (H \mathbf{a} + K \mathbf{b} + L \mathbf{c})\]

\[m \sim 0.1 \mu_B\]
**L-variable**

The four flux-pair configurations transform under $E_u$ irreducible representation.

Under point group operations of the tetragonal lattice $L=(L_x, L_y)$ behaves as a (polar) planar vector and it transforms under $E_u$ irreducible representation.

In our analysis we represent the state of the L-variable (loop current variable) by an an in-plane polar vector $L$.

The vector $L$ is odd under space inversion and time reversal but it is even under their product. Other common names for such situation are toroidal or anapole (Zeldovich, 1961) moment.
Observation of magnetic order in a superconducting \textit{YBa}_2\textit{Cu}_3\textit{O}_6.6\ single crystal using polarized neutron scattering

H. A. Mook,\textsuperscript{1,*} Y. Sidis,\textsuperscript{2} B. Fauqué,\textsuperscript{2} V. Balédent,\textsuperscript{2} and P. Bourges\textsuperscript{2}
Nature of the enigmatic pseudogap state: novel magnetic order in superconducting HgBa$_2$CuO$_{4+\delta}$

Y. Li$^1$, V. Balédent$^2$, N. Barisić$^{3,4}$, Y. Cho$^{3,5}$, B. Fauqué$^2$, Y. Sidis$^2$, G. Yu$^1$, X. Zhao$^{3,6}$, P. Bourges$^2$, M. Greven$^{3,7}$

(2008)
Kerr rotation is controlled by (antisymmetric part of) dielectric tensor. Equivalent to axial vector.

In ferromagnet the role of such axial vector is taken by magnetic moment.
Thermodynamic signature of a phase transition to the pseudogap phase of $YBa_2Cu_3O_x$ high-$T_C$ superconductor

B. Leridon, P. Monod, and D. Colson

(2008)
Comment on "Magnetic Order in the Pseudogap Phase of High-$T_c$ Superconductors"

Jeff E. Sonier

YBCO

![Graph showing temperature vs. hole doping for YBCO with different markers for $\mu$SR, Kerr effect, and neutrons.]
Cuprates. L-variable.

Global ordering of L-variables. Symmetry breaking.

Formation of (unordered) local flux pairs in each unit cell.

L-variable aka “flux-pair” aka “circulating current variable”

\[ T^* \] “strange metal” (MFL)

\[ T_c \]

\[ \sim 1eV \]

pseudogap
Basis of 4 states of L-variable

\[ \lambda_1 = \psi_1 - \phi_1 + \phi_2 \]
\[ \lambda_2 = \psi_2 - \phi_2 + \phi_3 \]
\[ \lambda_3 = \psi_3 - \phi_3 + \phi_4 \]
\[ \lambda_4 = \psi_4 - \phi_4 + \phi_1 . \]

\[ H_{flux} = \sum_r f(\lambda_1(r), \lambda_2(r), \lambda_3(r), \lambda_4(r)) \]

\[ |\theta_1\rangle = \begin{cases} + & \text{if } \theta_1 = 45^\circ \\ - & \text{if } \theta_1 = 135^\circ \end{cases} \]

\[ |\theta_2\rangle = \begin{cases} - & \text{if } \theta_2 = 135^\circ \\ + & \text{if } \theta_2 = 45^\circ \end{cases} \]

\[ |\theta_3\rangle = \begin{cases} - & \text{if } \theta_3 = 225^\circ \\ + & \text{if } \theta_3 = 135^\circ \end{cases} \]

\[ |\theta_4\rangle = \begin{cases} + & \text{if } \theta_4 = 315^\circ \\ - & \text{if } \theta_4 = 225^\circ \end{cases} \]

\[ |\theta_1\rangle = \begin{cases} + & \text{if } \theta_1 = 45^\circ \\ - & \text{if } \theta_1 = 135^\circ \end{cases} \]

\[ H = e^{i\phi_0} e^{i\psi_0} \leftrightarrow \text{h.c.} \]
To manage L-variables we introduce two operators

Operator L is used to define the (loop-current) states

\[ L |\theta\rangle_r = e^{i\theta_r} |\theta\rangle_r \]

The flipping operator U is used to induce transitions between the four states

\[ U |\theta\rangle_r = e^{i\pi/4} |\theta + \pi/2\rangle_r \quad U = e^{i\pi/4} \sum_{\theta=0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}} |\theta + \pi/2\rangle \langle \theta |. \]

\[ L = e^{i\pi/4} \begin{bmatrix} e^{i0} & 0 & 0 & 0 \\ 0 & e^{i\pi/2} & 0 & 0 \\ 0 & 0 & e^{i\pi} & 0 \\ 0 & 0 & 0 & e^{i3\pi/2} \end{bmatrix} \quad U = e^{i\pi/4} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \]
Classical system. Ashkin-Teller model.

$L$ is an operator in terms of which the 4 classical states of $L$-variable are defined. The configurational energy follows from the Hamiltonian:

$$H = \frac{1}{2} \sum_{<r,r'>} J_2 L(r)^\dagger L(r') + J_4 L(r)^\dagger L(r')^\dagger L(r'^\dagger) + h.c.$$

or

$$A = \sum_{<r,r'>} J_2 \cos(\theta_r - \theta_{r'}) + J_4 \cos(2\theta_r - 2\theta_{r'})$$

$$+ \sum_r h_4 \cos 4\theta_r$$

Ordering transition.

No specific heat singularity (negative $\alpha$) in a range of $J_2, J_4$

Four states can also be parameterized by a pair of (classical) Ising variables $\sigma, \tau$

$$H = - \sum_{<r,r'>} J_2 [\sigma_r \sigma_{r'} + \tau_r \tau_{r'}] + J_4 \sigma_r \tau_r \sigma_{r'} \tau_{r'}$$

(Jose, Kadanoff, Kirkpatrick, Nelson, 1977)
Quantum fluctuations.

Quantum fluctuations are described in terms of “flipping” operator -- $U$

\[
U | \phi \rangle_r = e^{i\phi_r} | \phi \rangle_r
\]

\[
\phi = [45^\circ, 135^\circ, 225^\circ, 315^\circ].
\]

\[
H = E (L + L^\dagger) + \sum_{<r,r'>} J_2 U_r^\dagger U_{r'} + J_4 U_r^\dagger U_{r'}^\dagger U_{r'} U_{r'} + \text{h.c.} + \text{dissipation term with a bath of FS electrons}
\]

\[
S = \frac{1}{E} \left( \frac{d\phi_r}{d\tau} \right)^2 + \sum_{<r,r'>} J_2 \cos(\phi_r - \phi_{r'}) + S_{\text{diss}} + J_4 \cos(2\theta_r - 2\theta_{r'}) + h_4 \cos 4\phi_r
\]

follows from special form of coupling to fermions
Quantum-critical fluctuations \((\alpha = 1)\)

\[
\chi(q, \omega) = \langle U_r(\tau) U_r^\dagger(\tau') \rangle_{q,\omega} = \langle e^{-i\phi_r(\tau)} e^{i\phi_r(\tau')} \rangle_{q,\omega}
\]

\[
\text{Im}\chi(q, \omega) = \text{th} \frac{\omega}{2T}, \quad |\omega| < \omega_0
\]

Coupling to the bath softens the stiffness of L-variables.

Aji, Varma (2007)
Coupling of L-variable to fermions on FS.

FS fermions couple to flipping operator \( U \). (c is an upper band electron)

\[
H_{coup} = \left( \begin{array}{c}
\text{single-particle} \\
\text{fermion operator}
\end{array} \right) (U + U^\dagger)
\]

\( J(rr') \) is a link current operator on the lattice. (d,p are lattice copper/oxygen electrons)

\[
j_{rr'} = id_r^\dagger p_{r+\hat{x}} + p_{r+\frac{\delta_x}{2}}^\dagger d_{r+\delta_x} + h.c.
\]

The operator that couples to \( U \) is a lattice link current circulation

\[
C = j(r, r + \hat{x}) + j(r + \hat{x}, r + \hat{x} + \hat{y}) + j(r + \hat{x} + \hat{y}, r + \hat{y}) + j(r + \hat{y}, r).
\]
Coupling of L-variable to fermions on FS.

To obtain the coupling we project the operator $C$ onto the upper band electrons and go to momentum space

$$H_{coup} = \sum_{k,k',\sigma} \left( \gamma_{k,k'} c_{\sigma k}^\dagger c_{\sigma k'} \right) (U + U^\dagger)_{k-k'}$$

$$\gamma_{k,k'} = i \gamma \left[ s_x(k + k') s_y(k - k') - s_y(k + k') s_x(k - k') \right] S_{xy}(k,k')$$

Omitting (lattice) details, the (symmetry) structure of coupling in momentum space is

$$\gamma_{k,k'} \propto i (k - k) \times (k + k') \propto i k \times k'$$

$$H_{coup} \sim \nabla \times j (U + U^\dagger)$$

The coupling vanishes in the long-wavelength limit, $q=(k-k')$

$$\gamma_q \propto i q \times k$$

$$S_{diss} \sim \sum_{\omega_n} \sum_{r} \alpha |\omega_n| q^2 |\phi_r(\omega_n)|^2$$

this is the origin of the factor $q^2$
Superconducting pairing symmetry.

Superconducting pairing is mediated by exchange of a local fluctuation.

\[ \Lambda(k, k') = \gamma(k, k') \gamma(-k, -k') \text{Re} \chi(\omega = \epsilon_k - \epsilon'_{k'}) . \]

Symmetry-wise:

\[ \Lambda(k, k') \propto -(k \times k')^2 \propto -[1 - \cos(2\theta - 2\theta')] . \]

On the (tetragonal) lattice:

\[ (k \times k')^2 = \frac{1}{2} \left[ (k^2_x + k^2_y)(k'^2_x + k'^2_y) - (k^2_x - k^2_y)(k'^2_x - k'^2_y) - (2k_x k_y)(2k'_x k'_y) \right] . \]

\[ \Lambda(k, k') = \lambda_0 \left[ \frac{F_s(k)F_s(k')}{N^2_s} - \frac{F_{d1}(k)F_{d1}(k')}{N^2_{d1}} - \frac{F_{d2}(k)F_{d2}(k')}{N^2_{d2}} \right] . \]