Collective modes and transport in Weyl semimetals

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### Life in the time of Topologitisis

(taken from A. Schnyder et al. in Proceedings of the L.D.Landau Memorial Conference "Advances in Theoretical Physics", June 22-26, 2008, Chernogolovka, Moscow region, Russia)

**TABLE 2. Summary of the main result of this paper.** Listed are again the ten symmetry classes of single particle Hamiltonians (from TABLE 1) classified in terms of the presence or absence of time-reversal symmetry (TRS) and particle-hole symmetry (PHS), as well as sublattice (or "chiral") symmetry (SLS) [17, 18, 19]. The last three columns list all possible topologically non-trivial quantum ground states as a function of symmetry class and spatial dimension $d$. The symbols $\mathbb{Z}$ and $\mathbb{Z}_2$ indicate that the space of quantum ground states is partitioned into different topological sectors labeled by an integer ($\mathbb{Z}$), or a $\mathbb{Z}_2$ quantity (two sectors only), respectively.

<table>
<thead>
<tr>
<th>System</th>
<th>Cartan nomenclature</th>
<th>TRS</th>
<th>PHS</th>
<th>SLS</th>
<th>$d=1$</th>
<th>$d=2$</th>
<th>$d=3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>standard (Wigner-Dyson)</td>
<td>A (unitary)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>$\mathbb{Z}$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>AI (orthogonal)</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>AI (symplectic)</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>chiral (sublattice)</td>
<td>AIII (chiral unit)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$\mathbb{Z}$</td>
<td>-</td>
<td>$\mathbb{Z}_2$</td>
</tr>
<tr>
<td></td>
<td>BDI (chiral orthog.)</td>
<td>+1</td>
<td>+1</td>
<td>1</td>
<td>$\mathbb{Z}$</td>
<td>-</td>
<td>$\mathbb{Z}_2$</td>
</tr>
<tr>
<td></td>
<td>CI (chiral sympl.)</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>$\mathbb{Z}_2$</td>
<td>-</td>
<td>$\mathbb{Z}_2$</td>
</tr>
<tr>
<td>BdG</td>
<td>D</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>$\mathbb{Z}_2$</td>
<td>$\mathbb{Z}$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>-</td>
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<tr>
<td></td>
<td>DIII</td>
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<td>CI</td>
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<td>1</td>
<td>-</td>
<td>$\mathbb{Z}_2$</td>
<td>-</td>
</tr>
</tbody>
</table>

**Newer* stuff:** gapless topological phases

*) circ. 1930-40s
No epidemic plateau seen yet...

APS MM abstracts

2008 2009 2010 2011 2012 2013 2014

(BiTe) (Majorana?)
Weyl semimetals are off to a good start
Weyl Semimetal (WS) is a collection of nondegenerate band touchings in 3D k-space

(Herring’37, Abrikosov&Beneslavsky’71, Wan et al’11, Burkov&Balents’11)

Two bands $\Rightarrow$ the spin degeneracy is lifted, either by T or I breaking
Where does one get a WS? (in a lattice system)

Ir-based pyrochlores: (Wan, Turner, Vishwanath&Savrasov’11)

IrO$_6$ polyhedra:

$\text{Ln}_2\text{Ir}_2\text{O}_7$:

\[ \text{LDA+U} \]

TI-based heterostructures: (Burkov&Balents’11)
Where does one get a WS? (continued)

Recently: $\text{Cd}_3\text{As}_2$ is a Dirac semimetal?


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**FIG. 2:** (Color online) Band dispersions and band-splitting in the plane passing through Dirac point $(0,0,k_2^c)$ and perpendicular to $\Gamma$-$Z$ for structure II. The $k$-points are indicated in cartesian coordinates. $X$ and $k_2^c$ are around 0.1 and 0.032 Å$^{-1}$, respectively.
Weyl semimetals (WS) are gapless phases with nontrivial topology

(review: Turner&Vishwanath, cond-mat: 1301.0330)

1. Nodes are stable due to topology, not symmetry (as, say, in graphene)
2. There are protected surface states (‘arcs’), which cannot be realized in a 2DEG.
3. Hall response is determined by the distance between nodes, and nothing else.
4. The chiral anomaly is also a topo. response
The chiral anomaly is the nonconservation of valley charges in the presence of EM fields.

1D Example: \[ \cdots \cdots \cdots \cdots \cdots x \]

Apply \[ E = -\frac{\partial A}{\partial t} \]

\[
(\partial_t \mp \partial_x) \Psi_{R,L} = 0
\]

yet

\[
\dot{N}_R - \dot{N}_L = \frac{e}{\pi \hbar} E
\]

(!)
In the 3D case, the B-field reduces the problem to a collection of 1D ones

$$\vec{B} = (0, 0, B),$$
$$H = \pm v \left[ \vec{\sigma}_{\perp} (\vec{p}_{\perp} - e\vec{A}) + \sigma_z p_z \right]$$

\[\Rightarrow\]

\[E_{n \neq 0}^{R,L}(p_z) = \pm v \sqrt{2|n|eB\hbar/c + p_z^2}
E_0^{R,L}(p_z) = \pm vp_z, \quad n = 0\]

\[\vec{\sigma}_{\perp} \quad \text{“graphene”} \quad p_z \quad \text{“gap”} \]

\[\dot{N}_R - \dot{N}_L = \frac{e^2}{2\pi^2\hbar^2 c} \mathbf{E} \cdot \mathbf{B} \quad \text{“3D chiral anomaly”}\]
Practical problem: symmetry-wise, WS are no different from more conventional phases, thus exhibit the same responses, in principle.
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How does one detect them then? There are a few ways out of this complication:

1) Look for unusual magnitude of effect

2) Unusual sign (Son, Spivak, 2012 negative "classical" magneto-resistance)

3) Unusual parameter dependence. (Parameswaran et al. 2011, "quantum-critical" conductivity, $\sigma(\omega) \propto \max(\omega, T)$)
Q: How to distinguish a WS from a small-gap semiconductor? Both in principle, and in practice.

A: The chiral anomaly [1] is a good candidate. It provides a nonlocal transport signature. The latter is absent in the usual small gap semiconductors.

How to “see” the chiral anomaly:

Proposals for crystalline systems:

- via anisotropies in the conductivity
  (Nielsen&Ninomiya, 1983; Aji, 2012)

- via large “classical” magnetoresistance
  (Son&Spivak, 2012)

- via magnon-plasmon coupling
  (Liu,Ye&Qi, 2012)
We would like to have an easily observable effect that does not exist without the chiral anomaly.
We will use the chiral anomaly to generate and detect nonlocal voltages, sensitive to magnetic field magnitude and direction.
In the presence of a magnetic field, transport current generates valley imbalance.
Inverse effect: In the presence of a magnetic field **AND** valley imbalance, there is a top-bottom voltage
Summary so far:

1. Four-terminal nonlocal geometry can help detect electric signals due to the presence of the chiral anomaly:
   \[
   \frac{|V_{nl}(x)|}{V_{SD}} \propto e^{-x/\ell_v}, \quad \ell_v = \sqrt{D \tau_v} \gg d
   \]

2. Should employ either tunneling leads, or this:

3. Locally applied B-fields are preferred

4. Detectors can be made non-invasive, if they are not too long
Transport theory = bulk transport equations + boundary conditions

The currents include the chiral modes contributions:

\[ j_{R,L} = -\frac{e}{\sigma} \nabla \mu_{\text{ec}}^{R,L} \pm \frac{e^2 B}{4\pi^2 \hbar^2 c} \mu^{R,L} \]

\[ \mu_{\text{ec}}^{R,L} = \mu^{R,L} + e\phi \]

(Vilenkin, 1980)

The continuity equations include the anomalous divergences:

\[ \nabla \cdot j_{R,L} + \partial_t \rho_{R,L} = \pm \frac{e^3}{4\pi^2 \hbar^2 c} E \cdot B \]

The final stationary transport equations contain only \( \mu_{\text{ec}}^{R,L} \)

\[ -\frac{\sigma}{e} \nabla^2 \mu_{\text{ec}}^{R,L} \pm \frac{\beta}{\sigma} \hat{n} \cdot \nabla \mu_{\text{ec}}^{R,L} = \pm \frac{e\nu_{3D}}{2\tau_v} \left( \mu_{\text{ec}}^{R} - \mu_{\text{ec}}^{L} \right) \]

\[ \beta = \frac{1}{2\pi\ell_B^2} \frac{e^2}{\hbar} \]
Boundary conditions: The simplest set of physically sound ones would suffice here.

Top surface:

\[ j_z^R (d) = \frac{g}{e} (\mu_{ec} (d) - \mu_S) + \frac{\beta}{e} \mu_{ec} (d), \]

\[ j_z^L (d) = \frac{g}{e} (\mu_{ec} (d) - \mu_S) - \frac{\beta}{e} \mu_S, \]

Bottom surface:

\[ j_z^R (0) = \frac{g}{e} (\mu_D - \mu_{ec} (0)) + \frac{\beta}{e} \mu_D, \]

\[ j_z^L (0) = \frac{g}{e} (\mu_D - \mu_{ec} (0)) - \frac{\beta}{e} \mu_{ec} (0). \]

Assumption: no inter-valley scattering under a lead.
The nonlocal voltages reach afar, and depend on the orientation of the B-fields

\[
\frac{V_{nl}(x)}{V_{SD}} = -\text{sign}(B_g)\text{sign}(B_d) \frac{\beta_d}{2g_d + \beta_d} \frac{\beta_g}{2g_g + \beta_g} e^{-\frac{|x|}{\ell_v}}
\]

\[
\beta_{g,d} = \frac{1}{2\pi \ell_B^{2}} \frac{e^2}{\hbar} \propto B_{g,d}, \quad \ell_v = \sqrt{D\tau_v} \gg d
\]
Tunneling contacts are beneficial, since leads act as shorts between valleys.

\[ g_{\text{contact}} \gg \sigma_{\text{bulk}}/d : \]  Good contact; bad for imbalance generation. Effective relaxation time can be as short as the diffusion time through the bulk.

\[ g_{\text{contact}} \ll \sigma_{\text{bulk}}/d : \]  Poor contact; good for imbalance generation. Intervalley scattering relaxes imbalances. Means almost no voltage drop across the bulk.
Local B-fields (i) reduce intervalley scattering, (ii) provide additional sensitivity to the C.A.

In B-field, open surfaces must induce intervalley scattering

Up-moving chiral mode cannot back-scatter into the same valley.
Local B-fields (i) reduce intervalley scattering, (ii) provide additional sensitivity to the C.A.
When is the detector non-invasive?

\[ j_{\text{leak}} \sim \Gamma_{\text{leads}} L_d \delta \mu, \quad \Gamma_{\text{leads}} \sim \frac{D}{d^2} \frac{g_{\text{contact}}}{\sigma/d} \]

\[ j_v \sim D \delta \mu / \ell_v \]

\[ j_{\text{leak}} \ll j_v \iff L_d \ll \frac{\sigma/d}{g_{\text{contact}}} \frac{d^2}{\ell_v}. \]

This condition is not very restrictive, since \( d \ll L_d \ll \ell_v \) is well within the applicability range of the theory.
Does chiral anomaly manifest itself in the spectrum of collective modes?

I. Panfilov, A. Burkov, DP Phys. Rev. B 89, 245103 (2014) (see also Son&Spivak’12)

Are there any new modes? “Hydrodynamics plasmons”?

(“yes” - Son&Spivak’12)
“Hydrodynamic plasmons”? No

Equations of motion with a self-consistent electric field:

\[ \nabla \cdot \mathbf{j}^{R,L} + \partial_t \rho^{R,L} = \pm \frac{e^3}{4\pi^2 \hbar^2 c} \mathbf{E} \cdot \mathbf{B} \]

\[ \mathbf{j}^{R,L} = -\frac{\sigma_D}{e} \nabla \mu_{ec}^{R,L} \pm \frac{e^2 \mathbf{B}}{4\pi^2 \hbar^2 c} \mu^{R,L}. \]

\[ \text{div} \mathbf{E} = 4\pi (\rho^L + \rho^R), \quad \rho^{R,L} = e\nu \mu^{R,L}. \]
"Hydrodynamic plasmons"? Only for large B.

Dispersion relation:

\[ \omega^2 + 8\pi i\sigma_D \omega - \frac{8\pi B^2}{\nu_B} \left( \frac{e^2}{4\pi^2 \hbar^2 c} \right)^2 = 0. \]

Solutions with a real part exist if

\[ \frac{8\pi B^2}{\nu_B} \left( \frac{e^2}{4\pi^2 \hbar^2 c} \right)^2 > (4\pi \sigma_D)^2. \]

It is unlikely to satisfy this condition for

\[ \frac{\hbar \nu}{\ell_B} \ll T, \mu \]
Undoped case: WS becomes compressible at any finite B-field

\[
\frac{\hbar v}{\ell_B} \gg T, \mu:
\]

\[
\omega_p^2 = v^2 q^2 + \frac{r_S}{\epsilon + \#r_S \ln \left( \frac{W}{\hbar v/\ell_B} \right)} \frac{2}{\pi} \frac{\hbar^2 v^2}{\ell_B^2}
\]

This translates into a non-analytic in B correction at finite doping:

\[
\omega_p^2 = j \propto (\mu_R - \mu_L)B
\]

Unitary limit, does not depend on e

\[
\text{E}
\]

\[
\text{B}
\]
1. Weyl semimetals are gapless topological phases

2. The Adler-Bell-Jackiw (chiral) anomaly provides a topological response.

3. In clean samples this response is measurable via a simple nonlocal electric measurement.

4. The chiral anomaly leads to new collective modes. The plasmon analog of the first sound does not exist at small magnetic fields though.